A Note on Mixed Duopolistic Competition in Higher Education Markets

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A higher education market is considered, the demand side of which consists of a continuum of individuals striving for utility maximization, and the supply side, of two universities aiming to optimally satisfy their preferences. Within a mixed duopoly setup, which embodies one public university and its counterpart, the private university, imperfect competition takes place. The public institution aims human capital maximization, whereas the private strives for profit maximization. I focus on two classical types of competition, namely Cournot and Stackelberg. Assuming that universities compete in admission standards, I see that the public university is more selective under the Cournot competition scenario than under the Stackelberg one. In contrast, I observe that the private university is similarly selective across the two competition scenarios. In line with the given heterogeneity in preferences, I notice that the public institution is always more selective than the private one, which essentially accepts all applications. Ultimately, I prove that social welfare produced under both types of competition is identical.

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Keywords: Mixed duopoly, Students' ability, Tuition fee, Cournot competition, Stackelberg competition.

1. INTRODUCTION

In spite of being a sound rationale for the economic modeling and provision for the vast majority of consumer goods, perfect competition models hardly provide any sound insight into educational provision. That is, HE sector hardly qualifies for any of the necessary criteria for the adoption of the perfect competition approach (Leslie and Johnson, 1974). Furthermore, HE markets are very segmented and differentiated with respect to many features. Clotfelter (1999) clearly illustrates this segmentation for US, where elite universities compete on a national basis for students, while less prestigious institutions compete on a regional or local basis. Moreover, Becker and Round (2009) by taking into consideration the so called dimensions of the market (i.e. product, geography, time) indicate that HE usually encompasses a number of singular markets. To give an illustration, Harvard University most likely will not directly compete for the same pool of students with Washington State University. And this simply occurs because of the crude fact that Harvard’s product-mix is to some extent different from that of Washington, institutions are located in different states, and production technology due to institutional and governmental regulations is not that flexible such as to allow for a direct strategic interaction between the two institutions.

As matter of fact, theoretical modeling of HE yet remains scarce. Among other contributions, I would like to highlight the following in particular. Rothschild and White (1995) develop a pricing model. In their proposed setup, students are at the same time input and output. The final output (human capital) is generated through accepting a variety of inputs (groups of different types of students). Instead of introducing a strategic interaction, the authors consider the production of human capital as a centralized process. Therefore, the whole problem is simplified into a social allocation function from which the maximization of human capital over the expenditures is analyzed. In the other side, Romero and Del Rey (2004) analyze the competition between public and private universities through a sequential decision for optimal quality, fees and admission exams. They compare the mixed duopoly results with the public monopoly benchmark, continuing with the comparative statics about social welfare. In a similar fashion, Romero (2005) analyzes competition among universities under monopolistic settings. In her model, universities are either public or private, and there is a sharp division between their objectives. That is, public aims social surplus maximization, while private aims profit maximization. Finally, Epple et. al. (2006) propose a general equilibrium model. The latter is akin to a perfect competition model in which consumers are differentiated by income and ability, they have homogeneous preferences and share perfect information. In the supply side, there are N-universities, which aim at quality maximization through enhancement of students’ body peer-effects, and instructional expenditures per student. As the same authors recommend, their proposed model is more appropriate to explain competition between private colleges.

I propose an alternative approach to deal with competition in these markets. In this regard, I attempt to take into account the aforesaid intrinsic features of higher education sector with respect to market fragmentation and product differentiation. In other words, I will follow the view which considers imperfect competition as an appropriate setup to analyze such types of markets.

More specifically, I build upon the basic framework provided in Romero (2005). However, here I only focus on a specific scenario of the mixed duopolistic setup since I consider quality and prices as exogenous. In other words, the approach intends to shed light on a scenario, where quality and fees are strongly regulated by government. In this regard, the approach becomes more simplified than that proposed by Romero and Del Rey (2004), as it significantly relaxes the extent of vertical differentiation by making it exclusively endogenous to admission standards. Although such a setup might initially sound very theoretical, it is not that far from realistic realms. With respect to Epple, et al. (2006), my approach will attempt to add more straightforwardness and simplicity. Also, I believe that the proposed setup will enrich that of Rothschild and White (1995) by introducing strategic interaction into the production process. Ultimately, through employing IO classical competition models such as Cournot and Stackelberg, a primary role to the strategic interaction between supply and demand is devoted.

The rest of the article is organized as follows: Section 2 outlines the preferences of individuals and monopolies represented by universities. Section 3 describes the general design of games. In Section 4, I show the Cournot scenario, solving the game and demonstrating the equilibrium. In Section 5 reports the Stackelberg scenario its equilibrium. In Section 6, I compare the two scenarios and I interpret the results in terms of social welfare. Finally, Section 7 contains results and discussion.

2. THE MODEL

I assume that quality standards and fees are strictly fixed by government directives. Therefore, the market, which I exclusively refer to a structure of strategic interaction, will be only composed two groups of players, i.e. individuals (i.e. potential students), and universities. The demand consists of individuals striving for utility maximization, whereas the supply, of two universities (i.e. one public plus one private) aiming to optimally satisfy their preferences.

2.1. Individuals

In line with the model proposed from Romero (2005), the economy consists of a continuum of individuals of measure one.

Individuals obtain utility, according to the following linear function:

$$u_i^j = w_i - f_j + h_i \quad \text{where } j = \{b, v\} \qquad \{1\}$$

where i-denoted variables are individual characteristics, and *j*-denoted variables are institutional characteristics. The u_i^j represents the utility of individual-*i* attending the higher institution- *j*. The w_i is the individual initial endowment – the sum of money she/he possesses in the very beginning. The f_j is the tuition fee charged by universities. Finally, h_i is the accumulated human capital by the individual –*i*.

Human capital is defined by the following function:

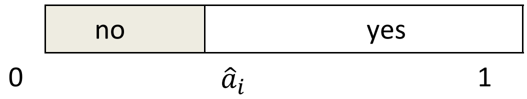
$$h_i = a_i Q_j \qquad \{2\}$$

concurrently, it depends on two variables: the student individual ability and the quality offered by the institution she/he attends.

Each variable in the model is continuous and uniformly distributed over the closed interval [0;1].

Also, I preserve the old setup of randomly distributed over a segment starting at 0 and ending at 1.

Figure 1: Indifference level



Let $\hat{a}_i$ be the ability of the student who is indifferent between attending university *j* and remaining uneducated, *i.e.*, $u_j^i = u_0^i$:

$$u_j^i = u_0^i \Longrightarrow w_i - f_j + h_i = w_i \Rightarrow h_i = f_j \Rightarrow a_i Q_j = f_j$$

$$\hat{a}_i = \frac{f_j}{Q_j} \qquad \{3\}$$

Those students of ability $a_i \geq \hat{a}_i$ are willing to attend university-*j,* while students of ability $a_i \leq \hat{a}_i$ prefer to remain uneducated.

2.2. The Mixed Duopoly

There are only two institutional players in the market, the public university and the private university that compete with each other in order to optimally satisfy their own preferences. Their utility functions represent their preferences. As the public aims maximization of consumer surplus, the private aims profit maximization. Formally, preferences are represented as follows:

$$\text{Public:} \quad U_b = \int_0^1 \int_{a_b}^1 (a_i Q_b - C(Q_b))\, da\, dw \qquad \{4\}$$

$$\text{Private:} \quad U_v = \int_0^1 \int_{a_v}^1 (f_v - C(Q_v))\ da\ dw \qquad \{5\}$$

These functions illustrate well the proper aim of each institution, in case of public, it is a continuous sum (double integral) of human capital created, minus the cost incurred to provide a certain level of quality. And in case of private, it is a continuous sum (double integral) of tuition fees minus the cost incurred to provide a certain level of quality. Bearing in mind that preferences also depend on ability level as the sum is generated by the continuum of the individuals admitted by each institution.

There is free access to the capital markets and no borrowing constrains are encountered. The latter state is represented with a zero lower bound in the integrals of the above functions.

Therefore, I have formally set a mixed duopoly framework – a situation of two entities with heterogeneous preferences and which compete for satisfying their utilities in the best feasible manner.

The two universities have the same cost structure, given by the following exponential relationship with qualities

$$C(Q_j) = Q_j^k, k > 1 . \qquad \{6\}$$

The cost function is increasing and convex in quality: $\frac{\partial C(Q_j)}{\partial Q_j} > 0$ and $\frac{\partial^2 C(Q_j)}{\partial^2 Q_j} > 0$.

The properties of the cost function indicate that the supply of teaching quality, i.e. highly professional academics, is not elastic enough, and thus we need to increase wages drastically in order to hire them. Taking into consideration that regulators may set the quality at certain level in the interval [0;1], the cost would also be fixed at a point within the interval [0;1].

3. DESIGN OF THE GAMES

I will analyze the mixed duopoly *equilibria* $(a_b^e,\ a_v^e)$ across the following scenarios:

1. One-shot simultaneous game or Cournot with perfect information;
2. Sequential game or Stackelberg leadership model, where one of the institutions has the advantage to move first and it is followed by the other. It is also designed with perfect information.

The strategic variable will be students' ability. Thus, institutions will compete only in ability level. Other variables, such as quality and tuition fee are considered exogenous or fixed by government regulation.

In the Cournot game, players choose their actions not knowing other firms' actions (Osborne, 2004). Similarly, we have the same good (service) produced and we have two players, which in our case, instead of firms, are universities. There is a difference in the preferences of the duopoly entities as they are not homogeneous as in the classical model, but it exist a degree of heterogeneity. The latter refers to the fact that the private university is profit oriented while the public one is human capital oriented. Moreover, demand function, instead of being down-sloping in prices, is uniformly distributed reflecting the uniformity of the abilities. The latter assertion is also justified by the fact that tuition fees (prices) and qualities are exogenous. These assumptions will transform the solving procedure into a more straightforward one.

Alternatively, in the sequential move game known as Stackelberg's duopoly game, I deal with a slightly different situation. Still, there are two players or universities, and public (private) knows about the action of the private (public) adjusting its decision according to the signal given by its "leader". Preferences are the same as in Cournot, with a small heterogeneity between players as it was aforementioned.

According to the literature, Stackelberg is an extensive game with finite horizon, so I employ backward induction technique to obtain the sub-game perfect equilibrium.

Obtaining *equilibria* of the both games, I discern what does happen with *equilibria* within and across scenarios. Ultimately, I examine those scenarios, which generate superior social welfare outcomes, other thing being equal.

4. THE COURNOT SCENARIO

In the simultaneous movement game, Nash equilibrium consists of a vector $(a_b^E;\ a_v^E)$, which satisfies the following system: $\{\frac{\partial U_b}{\partial a_b}=0; \frac{\partial U_v}{\partial a_v}=0\}$. In line with the classical model, I first derive the respective best response functions, namely BRF_b and BRF_v. More formally, $BRF_b = -a_b Q_b + C(Q_b) = 0$, and $BRF_v = -f_v + C(Q_v) = 0$. Solving the system composed by the best response functions, the equilibrium ability level for public is $a_b^E = \frac{C(Q_b)+f_v-C(Q_v)}{Q_b}$.

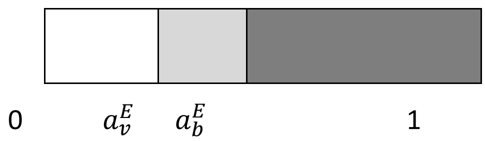
Considering that equilibrium ability level for private (i.e. a_v^E) cannot be derived from the system solution, I employ a different approach.

The private, in order to maximize its objective function, should set an ability level, which is shifted more on the left side (lower) rather than on the right side (higher), in accordance with the following inequality

$$\lim_{a_v \to 0} \int_0^1 \int_{a_v}^1 (f_v - C(Q_v))\ da\ dw \gg \lim_{a_v \to 1} \int_0^1 \int_{a_v}^1 (f_v - C(Q_v))\ da\ dw. \qquad \{7\}$$

The validity of such inequality is supported if I would consider a simplified form, i.e. $U_v = \big(f_v - C(Q_v)\big)(1-a_v)$, which is obtained through expanding the integral {5}. In order U_v to be maximized, a_v should take the lowest possible value, taking also into account that the term $(f_v - C(Q_v))$ is constant. But, we know that the lowest possible operational value is the indifference level {3}. This means that the private will not run exams, or in other words, it accepts all applications. Graphically, the market coverage for public and private universities can be illustrated as below.

Figure 2: Cournot equilibrium



Proposition 1: *In the simultaneous move case, the public university is more selective in student ability than the private.*

Since $f_v - C(Q_v) \geq 0$ the validity of the proposition is ensured. Otherwise, the private would not operate in the market. Moreover, given that a_b^E differs from the indifference level, i.e. $\frac{f}{Q}$, it should be located on its right side, if not, the public would not operate either. Thus, $a_b^E > a_v^E$, as depicted above.

5. THE STACKELBERG SCENARIO

Stackelberg duopoly game is a type of extensive game. Here, one of the firms, in our case universities, moves first. I assume that the private university takes the leadership of the ability level and thus decides first about its rationing level. Then the public university observes the leader's decision and takes the best possible action incorporating the information from the private.

In line with Osborne (2004), in order to solve a game with finite horizon, I employ the method of backward induction. First of all, I solve the maximization problem for public and later for private (leader) incorporating the best response function of public.

Public university will set an optimal level a_b^*, which maximizes its preferences $\frac{\partial U_b}{\partial a_b} = 0$. Performing the necessary calculations, I find out that $a_b^* = \frac{C(Q_b)}{Q_b}$ (see appendix). Subsequently, the private will set its optimal a_v^*. The value will fall in the shortened segment $(a_v; \frac{C(Q_b)}{Q_b})$ instead of the full segment $(a_v; 1)$. Consequently, the private will have to maximize the following preference function

$$U_v = \int_0^1 \int_{a_v}^{\frac{C(Q_b)}{Q_b}} (f_v - C(Q_v))\ da\, dw. \qquad \{8\}$$

As it is shown in appendix, the first order condition is $\frac{\partial U_b}{\partial a_v} = -\big(f_v - C(Q_v)\big) = 0$.

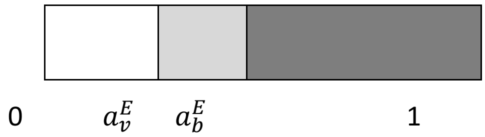
From the latter expression, one cannot get a_v^* value. For this reason, in order to maximize the market coverage, private will set a cut-off ability equal to the indifference level, i.e. $a_v^* = a_v^E = \frac{f_v}{Q_v} = \frac{f}{Q}$. Similarly to Cournot case, private will not run exams, i.e. it will simply accept all applications.

As the best response functions are scalars for both entities, the sub-game perfect equilibrium is located in the intersection of a horizontal line with a vertical line. Formally, $(a_b^*; a_v^*) = (a_b^E; a_v^E) = (\frac{C(Q_b)}{Q_b}; \frac{f}{Q})$.

In the inverse case, where public is leader and private follower, the same equilibrium vector is obtained. This means that Stackelberg *equilibria* are fully symmetric (see appendix). The equilibrium level of ability is shifted more on the right side for public university being more selective, while the private accepts all application covering the left part of the market.

Graphically, the distribution is illustrated as follows.

Figure 2: Stackelberg equilibrium



Proposition 2: *In the sequential movement case, the public university is more selective in student ability than the private.*

6. COMPARATIVE STATICS AND SOCIAL WELFARE

At this stage, one can further investigate shedding more light on comparative statics and welfare implication within and across the proposed competition setups. In order to clearly analyze the latter aspects, I construct the following table which summarizes all *equilibria*.

Table 1: Equilibrium differences

Equilibria		
Cournot Scenario	$\frac{C(Q_b)+f_v-C(Q_v)}{Q_b}$	$\frac{f_v}{Q_v}$
Stackelberg Scenario	$\frac{C(Q_b)}{Q_b}$	$\frac{f_v}{Q_v}$
Change	↘	ϕ

Source: Own Compilation.

As this table well illustrates, moving from the Cournot scenario to the Stackelberg one, the equilibrium level of cut-off ability falls for public, but it does not exhibit any change for private.

Proposition 3: *In the Cournot competition, the public university is more selective than in Stackelberg competition, on the other hand, the private is equally selective across scenarios.*

As concerns social welfare implications, I initially formalize social welfare with the following function

$$SW_D = Q - \int_{a^E}^{1} f da + \int_{a^E}^{1} h_i da = Q - F + Q \int_{a^E}^{1} a_i = Q\left(1 + \int_{a^E}^{1} da\right) - F. \qquad \{9\}$$

Since qualities and tuition fees are exogenous, it implies that all variables, but cut off abilities, are scalars. Also, as the vector ability a^E always corresponds to the minimal possible value, i.e. indifference level $\frac{f}{Q}$, a complete market coverage is guaranteed. Consequently, social welfare produced by duopoly under both scenarios is identical.

Proposition 4: *Total social welfare produced under both scenarios is identical.*

The result here suggests interesting insights for regulation, indicating that policies about competition which are aligned with my proposed setup should not necessarily affect welfare.

4. Results and Discussion

I have showed a duopolistic model in which two distinct universities, i.e. one private and another public, compete in order to assure the biggest market coverage. I was interested in investigating what occur to *equilibria* within the game and across games, moving from a Cournot competition scenario to a Stackelberg one.

First, within the game I notice that public is more selective than private. This can be justified with the fact of the presumed heterogeneity in preferences, where public is human capital oriented whereas private is profit oriented. Hence, for public, expectations are well aligned with *equilibria* outcomes, which clearly indicate its tendency to impose elevated admission standards.

In the other side, across scenarios, market is completely and equally covered. Furthermore, social welfare produced is also equivalent. I detect some variations in the first coordinate of the *equilibria* vector. More precisely, public university in Cournot competition is more selective than in Stackelberg competition and I justify it by acknowledging the uncertainty in Cournot, and the need for public to become more selective in order to avoid exit from the market.

I believe that this note proposes interesting hints to regulators for policies that can be undertaken on the subject of higher education and its regulation. At the same time, I am aware of the shortcomings. The simplification of variables to the unity magnitude and their continuity feature do not always hold. Also, the extreme heterogeneity in universities' preferences may not always be aligned with the reality. However, I deem this to be a very first step toward a more extended model, which can also be testable with empirical data.

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APPENDICES

Solution of the Cournot game

A vector $(a_b^E;\ a_v^E)$, which solve the system of the best response functions: $\left\{\frac{\partial U_b}{\partial a_b}=0;\frac{\partial U_v}{\partial a_v}=0\right\}$, should be found. First, let me expand the public preferences expressed in an integral function into an easier derivable form:

$U_b = \int_0^1\int_{a_b}^1(a_iQ_b - C(Q_b))\,da\,dw = \int_0^1(\int_{a_b}^1 a_iQ_b da - \int_{a_b}^1 C(Q_b)da)dw = \int_0^1\left(Q_b\frac{a_i^2}{2}\,I_{a_b}^1 - C(Q_b)a_iI_{a_b}^1\right)dw = \int_0^1\left(Q_b\frac{(1-a_b^2))}{2} - C(Q_b)(1-a_b)\right)dw = \left(Q_b\frac{(1-a_b^2)}{2} - C(Q_b)(1-a_b)\right)wI_0^1 = Q_b\frac{(1-a_b^2}{2} - C(Q_b)(1-a_b)$.

Then, the expanded function is: $U_b = Q_b\frac{(1-a_b^2)}{2} - C(Q_b)(1-a_b)$. Therefore, we calculate the first order condition as follows.

$$\frac{\partial U_b}{\partial a_b} = Q_b(-a_b) + C(Q_b) = 0 \Longrightarrow -a_bQ_b + C(Q_b) = 0 \Longrightarrow a_b^* = \frac{C(Q_b)}{Q_b}$$

$$\frac{\partial^2 U_b}{\partial^2 a_b} = -Q_b$$

Since the order condition is negative, it implies that U_b is concave. The best response function for the public is: $a_b = \frac{C(Q_b)}{Q_b}$.

Private preferences can be expanded as follows. $U_v = \int_0^1\int_{a_v}^1(f_v - C(Q_v))\,da\,dw = \int_0^1[\int_{a_v}^1 f_v da - \int_{a_v}^1 C(Q_v)da]dw = \int_0^1[f_va_iI_{a_v}^1 - C(Q_v)a_iI_{a_v}^1]dw = \int_0^1[f_v(1-a_v) - C(Q_v)(1-a_v)]dw = ([f_v - C(Q_v)(1-a_v)]wI_0^1 = (f_v - C(Q_v))(1-a_v)$

Thus, $U_v = \big(f_v - C(Q_v)\big)(1-a_v)$.

As optimal a_v must satisfy $\frac{\partial U_v}{\partial a_v} = -f_v + C(Q_v) = 0$.

Since the second order condition is negative, it implies that U_v is concave. Duopoly equilibrium is the intersection of the best response functions BRF_j for $j = \{b, v\}$

$$\begin{cases}\dfrac{\partial U_b}{\partial a_b} = -a_bQ_b + C(Q_b) = 0\\[2ex] \dfrac{\partial U_v}{\partial a_v} = -f_v + C(Q_v) = 0\end{cases}$$

$$-a_bQ_b + C(Q_b) = -f_v + C(Q_v) \Longrightarrow C(Q_b) + f_v - C(Q_v) = a_bQ_b$$

$$\Longrightarrow a_b^E = \frac{C(Q_b) + f_v - C(Q_v)}{Q_b}$$

Solution of Stackelberg with private leadership

A vector $(a_b^E; a_v^E)$, which solves the system of the best response functions $\left\{\frac{\partial U_b}{\partial a_b} = 0; \frac{\partial U_v}{\partial a_v} = 0\right\}$, should be found. Hence, I solve the game with the method of backward induction. First, I find the optimal a_b for the public (follower) which is the same as in the simultaneous game

$\frac{\partial U_b}{\partial a_b} = Q_b(-a_b) + C(Q_b) = 0 \Longrightarrow -a_b Q_b + C(Q_b) = 0 \Longrightarrow a_b^* = \frac{C(Q_b)}{Q_b}$.

Afterwards, given the public best response, the private (leader) has to choose a_v which maximizes the following preferences:

$$U_v = \int_0^1 \int_{a_v}^{\frac{C(Q_b)}{Q_b}} (f_v - C(Q_v))\ da\, dw$$

For a more straightforward derivation procedure, I simplify the above preferences as follows

$U_v = \int_0^1 \int_{a_v}^{\frac{C(Q_b)}{Q_b}} (f_v - C(Q_v))\ da\, dw = \int_0^1 (f_v a_i - C(Q_v) a_i) I_{a_v}^{\frac{C(Q_b)}{Q_b}}\, dw = \int_0^1 \left(a_i (f_v - C(Q_v)) I_{a_v}^{\frac{C(Q_b)}{Q_b}} \right) dw\ ) = \int_0^1 \left(\frac{C(Q_b)}{Q_b} (f_v - C(Q_v)) - a_v (f_v - C(Q_v)) \right) dw = \left((f_v - C(Q_v)) \left(\frac{C(Q_b)}{Q_b} - a_v \right) \right) w I_0^1 = (f_v - C(Q_v)) \left(\frac{C(Q_b)}{Q_b} - a_v \right)$.

Thus, the expanded U_v is $U_v = (f_v - C(Q_v)) \left(\frac{C(Q_b)}{Q_b} - a_v \right)$. So, first order condition is $\frac{\partial U_v}{\partial a_v} = -(f_v - C(Q_v)) = 0$.

Solution of Stackelberg with public leadership

A vector $(a_b^E; a_v^E)$, which solve the system of the best response functions $\left\{\frac{\partial U_b}{\partial a_b} = 0; \frac{\partial U_v}{\partial a_v} = 0\right\}$, should be found. I solve the sequential game with the method of backward induction. First, I find an optimal a_b for the private (follower).

The expanded preference function for private is $U_v = (f_v - C(Q_v))(1 - a_v)$. Since $(f_v - C(Q_v))$ is considered as constant, maximization of U_v occurs when the private sets lowest operational value which is the indifference level (i.e. $a_v^* = \frac{f_v}{Q_v}$). The public should take its decision incorporating the best decision taken by the private (follower). Hence, the public university will maximize the following utility function

$$U_b = \int_0^1 \int_{a_b}^{\frac{f_v}{Q_v}} (a_i Q_b - C(Q_b)) da dw$$

which is simplified as follows

$$U_b = \int_0^1 \int_{a_b}^{\frac{f_v}{Q_v}} \left(a_i Q_b - C(Q_b)\right) da dw = \int_0^1 \left(\int_{a_b}^{\frac{f_v}{Q_v}} a_i Q_b da - \int_{a_b}^{\frac{f_v}{Q_v}} C(Q_b) da \right) dw = \int_0^1 (Q_b \frac{a_i^2}{2} I_{a_b}^{\frac{f_v}{Q_v}}$$

$$- C(Q_b) I_{a_b}^{\frac{f_v}{Q_v}}) \, dw = \int_0^1 (Q_b \frac{\left(\frac{f_v^2}{Q_v^2} - a_b^2\right)}{2} - C(Q_b)(\frac{f_v}{Q_v} - a_b))dw$$

$$= (Q_b \frac{\left(\frac{f_v^2}{Q_v^2} - a_b^2\right)}{2} - C(Q_b)(\frac{f_v}{Q_v} - a_b))w \, I_0^1$$

$$= \frac{2f_v^2}{Q_v^2} Q_b - \frac{a_b^2}{2} Q_b - \frac{f_v}{Q_v} C(Q_b) + a_b \, C(Q_b)$$

Thus, the U_b in its simplified form is $U_b = \frac{2f_v^2}{Q_v^2} Q_b - \frac{a_b^2}{2} Q_b - \frac{f_v}{Q_v} C(Q_b) + a_b \, C(Q_b)$.

Then, first order condition is $\frac{\partial U_b}{\partial a_b} = -a_b Q_b + C(Q_b) = 0$. Therefore, the optimal a_b for the public having incorporated the information from the private is $a_b^* = \frac{C(Q_b)}{Q_b}$. The vector which includes the two optimal values is also the sequential *equilibria* of the market, i.e. $(\frac{C(Q_b)}{Q_b}; \frac{f_v}{Q_v})$.