4. Public expenditure on education and skill formation: are there simple rules to maximise skills?

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Abstract
The ratio of skill to unskilled labour stocks in the economy is widely acknowledged to have an important role for development. Can education policy affect the evolution of this ratio? This paper shows that it can, and it also shows that the actual effect of education policy depends on the allocation rule of the budget across educational levels. The consideration of a stylized hierarchical education model allows us to develop analytical conditions under which the allocation rule favours the accumulation of skills. The analysis has implication for policy makers in developing countries, where skill formation is much needed, because it shows that observed allocation rules usually violate the maximization condition by the assignment of higher-than-optimal resources to higher education.

Keywords: education budget, skills accumulation

JEL Classification: I21, I22, I28

1. Introduction

Even when the links between skills and growth, and hence education and growth, are well established on theoretical grounds (mainly in the endogenous growth theory, see for instance, Lucas, 1988, and Romer, 1986), the empirical evidence of these links is weak. To explain this, several authors (Birdsall et al., 1998; Gemmel, 1996; Papageorgiou, 2003) have stressed the importance of the distinction between the different stages of human capital creation for development and, therefore, the relevance of considering the internal allocation rules of the education budget.

The skill-to-unskilled stock ratio in developing and developed countries shows significant differences. According to UNESCO, the proportion of population with below upper secondary as maximum educational attainment of adult population in the OECD country members is, on average, 29% (data for 2005), whereas the proportion of lower secondary as maximum attainment in developing countries is much higher; for instance, in Brazil, the rate is 70.5%, and it is even higher in many African countries, with rates over 90%. Explanations to this gap are easy to find, as in developing countries, the accumulation of skills is hindered by inefficient education systems, often aggravating funding difficulties.

For this reason, the system structure matters because the performance at earlier stages affects the output at higher levels; the budget allocation rule should take this into account. To analyze this point, hierarchical education models have been used by Driskill and Horowitz (2002, 2009) and Su (2004). As noted by Su (2004), hierarchical structure in educational systems implies that levels are not perfect substitutes, which means that different allocation of similar budget size have different effects on aggregate efficiency and distribution. This paper discusses the allocation rules of the educational budget in a stylized two-level education system characterized by internal inefficiency. Dealing with inefficiency optimally will allow the achievement of a maximum ratio of skill to unskilled labour, considered the target. The simplicity of the model allows the development of analytical conditions under which the allocation rule favours the accumulation of skills and allows the development of several clear-cut policy recommendations that may serve as a guide to policy makers.

This article is organized as follows. Section 2 describes the current situation regarding education budget allocation and accumulation of skills. Section 3 presents the education model

and its properties. In Section 4, the conditions to maximize the stock of skills are discussed. The conclusion is presented in Section 5. An Appendix with mathematical details also is presented.

2. The current situation

The distribution of skills across countries varies considerably, especially between developed and developing countries (see Table 1). In developing countries, the majority of the population (above a half) has primary education or less as maximum educational attainment, whereas in developed countries, the proportion of the population with tertiary education is more than one third of the total population, and it is as high as 46% in the case of Canada.

Table 1. Educational attainment of the adult population. Distribution of the population aged 25 and older, by highest level of education attained (in percentages)

Country	Year	No schooling and primary complete or incomplete	Lower secondary	Upper secondary	Tertiary education
Argentina	2004	43.8	14.2	28.4	13.6
Bangladesh	2001	73.3	9.6	12.9	4.2
Botswana	2000	75.3	15.7	5.9	3.1
Brazil	2004	57.5	13.0	21.2	8.1
Chile	2004	24.0	26.0	36.9	13.2
Costa Rica	2007	50.9	13.8	18.5	15.0
India	2000	77.7	12.4	6.5	3.3
Kuwait*	2006	55.2	19.2	17.2	8.3
Mauritius*	2000	60.5	18.6	17.6	2.6
Mozambique	2000	96.9	2.3	0.8	0.1
Nigeria	2000	97.1	1.8	0.7	0.4
Uganda	2002	88.5	5.1	1.6	4.8
Uruguay	2006	52.8	22.4	15.1	9.6
Australia	2005	9.1	25.8	33.3	31.5
Canada	2004/2005	4.9	9.9	39.2	46.1
Finland*	2006	22.0	8.9	38.8	30.3
Ireland*	2006	23.7	16.3	31.2	26.4
Japan	2004/2005			60.1	40.0
New Zealand	2005		21.3	51.6	27.1
Republic of Korea	2005/2006	11.9	12.6	43.9	31.6
United Kingdom	2004/2005		14.4	55.9	29.6
United States	2005	6.3	8.5	49.0	36.2

Notes: Last data available. Total may differ from 100% because of missing information or rounding.* Upper secondary includes postsecondary nontertiary.
Source: Own elaboration with data from UNESCO/UIS WEI (www.uis.unesco.org/publications/wei2007); UNESCO, Global Education Digest 2009; Barro and Lee data set.

In many developing countries education systems show several weaknesses, especially in quality and coverage. In many cases, the expenditure on public education per student is far behind that in the developed world, but as argued by Birdsall et al. (1998), Gemmel (1996), and Papageorgiou (2003), both the size and the efficiency of the allocation of the public funds for education are relevant for the overall systemic performance.

A hint on allocation rules can be obtained by examining Table 2, which shows that, in general, although the government preferences do not differ very much across developing and developed countries in terms of the size of the budget (measured by the public expenditure on education as % of GDP, shown in the last column of the table), there are significant differences in the preference over the budget allocation (measured by the public expenditure per pupil as a % of GDP per capita by education level, shown in the first, second, and third data columns). As can be seen in Table 2, countries such as US and Japan have a perfectly flat allocation pattern (levels "equally preferred"), and in general, developed countries have a quite even distribution, with the exception of Korea, which allocates less than average to higher education. The situation among developing countries is more heterogeneous. For instance, whereas in countries, such as Chile and Argentina, the distribution is quite flat, there are many countries that display strong preferences for higher education, some of them extraordinary high, such as Mozambique and Botswana.

Considering the data in Tables 1 and 2, the observed differences in educational budget allocation rules and systemic performance (measured by the educational attainment of the population) lead to the question of the role of budget allocation in skills formation. For instance, as noted by Gemmel (1996), there is a key skill level for each development stage: human capital effects on growth are most evident at the primary level in low-income countries; for higher income developing countries, the key is the secondary level, whereas the tertiary level is the most relevant in developed countries. Thus, according to this approach, many African countries would be using allocation rules contrary to their development needs. In the long term, the disparity between skills endowment between the rich and poor countries is likely to widen, as richer countries are able to invest more money to expand and improve their educational services, generating virtuous or vicious circles

Table 2. Public expenditure per pupil as a % of GDP per capita by education level and public expenditure on education as % of GDP

	Public expenditure per pupil as a % of GDP per capita				Public expenditure on education as % of GDP
COUNTRY	Primary	Secondary	Tertiary	All levels	
Argentina	12	19	13	14	4
Bangladesh	9	15	46	13	2
Botswana	16	41	450	34	10
Brazil	14	12	34	15	4
Chile	12	13	13	13	3
Costa Rica	17	17	36	19	5
India	9	17	58	15	3
Kuwait	12	16	102	22	5
Mauritius	12	19	37	17	4
Mozambique	16	69	570	23	5
Nigeria	31	46	366	34	3
Uganda	11	32	179	14	5
Uruguay	8	10	18	11	3
Australia	17	16	24	18	5
Finland	18	32	35	28	6
Ireland	15	22	25	19	5
Japan	22	22	20	22	4
New Zealand	19	22	28	22	6
Republic of Korea	18	23	9	17	4
United Kingdom	20	25	30	24	5
United States	22	24	24	23	6

Note: Averages of available years 2004-2006.

Source: Own elaboration with data from UNESCO data base http://stats.uis.unesco.org/unesco/ReportFolders/ReportFolders.aspx

This article focuses on the role of budget allocation rules on the skill formation process. Under this approach, considering a desired target of skill share in labour produce, policy makers could allocate resources accordingly. The conditions to do this efficiently will be discussed in the rest of the paper.

3. The education model

As the learning process is cumulative, the indicator f_m is defined as $f_m = \sum_m q_j$, which is the knowledge accumulated per student who has completed up to level m, where q_j is the accumulation at j. The indicator f_m measures the human capital accumulated during the schooling process; the q_j accumulated at different levels are not perfect substitutes, so the allocation of resources across them will affect human capital accumulation.

A two-level education system is considered, consisting of basic and higher education ($j = B, H$), The output per student is $q_j = q_j(k_j)$, where k_j measures the resource intensity per student, and $\partial q_j/\partial k_j > 0$, $\partial^2 q_j/\partial^2 k_j < 0$, it is assumed to measure "school quality." Students leave the system early when the quality of education they receive is poor; the output per student is taken as the determinant of early dropouts, $\theta = \theta(q_B)$, where $\partial\theta/\partial q_B < 0$ and $\partial^2\theta/\partial^2 q_B < 0$.

The composition of the inflow of labour produced depends on time of exit and on school quality. Thus, the accumulation process is driven by

$$dL_U = \theta E_B f_B$$
$$dL_S = E_H f_H = (1-\theta) E_B f_H$$

where θ is the early exit rate, and dL_U and dL_S are the inflow of units of unskilled and skilled labour, respectively. The marginal ratio of skilled to unskilled labour produced can be defined as:

$$\xi(k_B, k_H) = \frac{dL_S}{dL_U} = \frac{1-\theta}{\theta}\frac{f_H}{f_B}$$

The ratio of skilled to unskilled labour in the economy is modified by ξ. When $\xi > L_S/L_U$, where L_S and L_U are the stocks of skilled and unskilled labour, respectively, the ratio of skilled to unskilled labour in the economy rises; it declines when $\xi < L_S/L_U$ and remains unchanged when growth is balanced

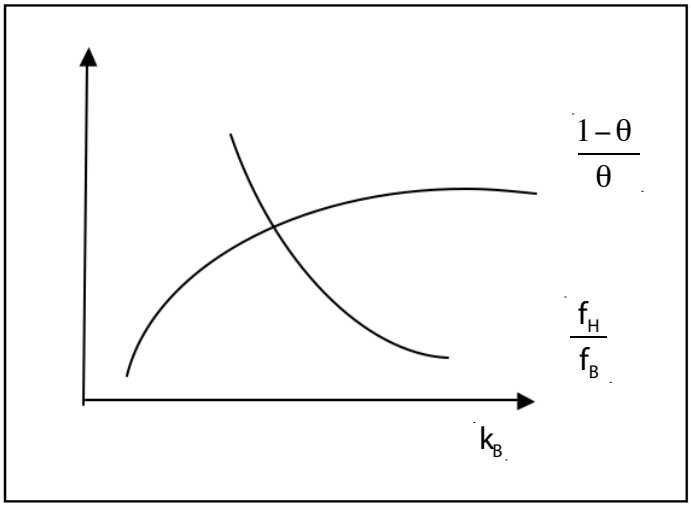
As the marginal ratio ξ is dependent on the capital intensity of the basic and high education, totally differentiating and after some manipulation results:

$$\hat{\xi} = \left(\hat{\frac{f_H}{f_B}}\right) + \frac{\hat{S}}{\theta}$$

where a hut (^) placed over the variables denotes rate of growth, and S is the survival rate defined as $S = 1-\theta$.

The evolution of ξ depends on the effects of allocation on the survival-to-exit rate and on the relative human capital accumulation across levels. This is presented in Figure 1, considering $\partial\left(\frac{1-\theta}{\theta}\right)\Big/\partial k_B > 0$ $\partial^2\left(\frac{1-\theta}{\theta}\right)\Big/\partial^2 k_B < 0$, $\partial(f_H/f_B)/\partial k_B < 0$, and $\partial^2(f_H/f_B)/\partial^2 k_B > 0$, which implies the variability of ξ over k_B.

Figure 1



Specific conditions allow to determine the sign of ξ. In general, $d\xi > 0$ when

$$\frac{d(f_H / f_B)}{f_H / f_B} > -\frac{d(1-\theta/\theta)}{(1-\theta)/\theta}$$

4. Properties and implications

The properties of ξ, the conditions under which $d\xi > 0$ and its implications, will be analyzed in what follows.

Property 1. $\partial\xi/\partial k_B > 0$ if $\varepsilon_{\theta\, q_B} > s_H(1-\theta)$, where $s_H = q_H / f_H$ and $\varepsilon_{\theta\, q_B} = -\partial\theta/\partial q_B \; q_B/\theta$. (See Appendix for Demonstration 1)

The capital intensity in basic education will have a positive effect on the marginal ratio of skilled to unskilled labour if the elasticity of the dropout variable to the quality of basic education (J_{N_H}) is high. So, for a given s_H, if the survival parameter is too low, the higher the possibility that the increase in capital intensity in basic education have a positive effect on the marginal ratio ξ.

Implication1. ξ is a non monotonic function of k_B. It follows from the demonstration that $\partial\xi/\partial k_B$ has an indeterminate sign.

Implication 2. ξ is a non monotonic function of K.

This can be seen by totally differentiating ξ:

$$d\xi = \frac{\partial\xi}{\partial k_H} dk_H + \frac{\partial\xi}{\partial k_B} dk_B$$

where considering that the sign of $\partial\xi/\partial k_B$ is indeterminate and that $\frac{\partial\xi}{\partial k_H} = \frac{1-\theta}{\theta}\frac{q_H'}{q_B} > 0$, coupled with that by definition $\partial k_j/\partial K > 0$, $j = B, H$, it follows that ξ is a non monotonic function of K.

Proposition 2. The allocation of more resources to basic education (with fixed budget and enrollment) will increase the marginal ratio of skilled to unskilled labour, that is, $d\xi/dk_B > 0$, if

$$\frac{\varepsilon_{\theta\, q_B}}{\varepsilon_{q_B k_B} + \varepsilon_{q_H k_H}\varepsilon_{k_H k_B}} > s_H(1-\theta)$$

where $\varepsilon_{\theta\, k_B} = -\partial\theta/\partial k_B \; k_B/\theta$, $\varepsilon_{h_H k_B} = -dk_H/dk_B \; k_B/k_H$ and $\varepsilon_{q_j k_j} = -\partial q_j/\partial k_j \; k_j/q_j$, $j = B, H$. (See the Appendix for Demonstration 2).

Implication 3. Considering that $k_B = B_B k$, where $B_B = K_B/K$, when $d\xi/dk_B > 0$ it also holds that $d\xi/dB_B > 0$, assuming k is constant.

5. Are there clear-cut policy recommendations?

For a given budget, it is possible to find a rule to maximize the skill to unskilled ratio of labour produced by maximizing $\xi(k_B, k_H)$ subject to $K = K_B + K_H$ and technology parameters. From the first order conditions, follows that

$$\frac{d\xi}{dk_B} = 0 \Leftrightarrow \frac{\varepsilon_{\theta k_B}}{\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}} = s_H(1-\theta)$$

where $\varepsilon_{\theta k_B} = -\partial\theta/\partial k_B \; k_B/\theta$, $\varepsilon_{h_H k_B} = -dk_H/dk_B \; k_B/k_H$, and $\varepsilon_{q_j k_j} = -\partial q_j/\partial k_j \; k_j/q_j$, $j = B, H$. The program has no closed solution, but some clear hints can be obtained. It can be shown that

$$\varepsilon_{\xi k_B} = \frac{\varepsilon_{\theta k_B}}{1-\theta} - s_H\left[\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}\right]$$

Considering also the "quasi-neutral" assumption on education technology that $\varepsilon_{q_B k_B} = \varepsilon_{q_H k_H} = \varepsilon_{q_j k_j}$, the above expression can be written as:

$$\varepsilon_{\xi k_B} = \frac{\theta}{1-\theta}\varepsilon_{\theta k_B}\left(\frac{1}{\theta} - s_H\varepsilon_{q_j k_j}\right) - \frac{s_H}{B_H}\varepsilon_{q_j k_j}$$

where $B_H = K_H/K$ is the participation of higher education in the total budget.

The determinants of the elasticity of ξ with respect to the resource intensity in basic education can be shown using the above expression. It shows that the elasticity of the marginal ratio of skilled to unskilled labour relative to the resource intensity in basic education is higher:

- The higher is $\varepsilon_{\theta k_B}$, the responsiveness of the early exit rate to the resource intensity.
- The higher is s_B ($s_B = 1 - s_H$), the contribution of basic education in total human capital accumulated.
- The lower is B_B ($B_B = 1 - B_H$), the participation of basic education in the total budget.

The level of the early exit rate (θ) has an ambiguous role. The former aspect listed is a pure technology parameter, the latter is a pure policy variable; the second value listed is a combination of technology and policy aspects. Note that the education technology plays a crucial role. For instance, in an extreme case, $\varepsilon_{\theta k_B}$ could be zero, in which case, the effect on ξ of an increase in k_B would be negative.

These results imply that in many developing countries with bad systemic outcomes due to the poor performance of basic education, the increase in the share of resources to basic education ($B_B = K_B/K$) could be more effective in terms of increasing the amount of skill labour in relation to unskilled.

6 ...an explanation to poverty traps?

Some countries seem to be lagging behind global pace of accumulation of skills. Is catching up possible? The poor performance of the education sector in many countries cast serious doubts on this. In most of developing countries the share of the population (aged 25 and over) which has primary education as the highest level of educational attainment is around a half or over, in the US the United Kingdom and most of developed countries this share is below 10%. In the other extreme, while the share of less developed countries speed had made a significant difference: for instance, according to Barro and Lee data set in 1960 the average years of schooling in Uruguay was 5.3, higher than Republic of Korea where it was 3.23; however, during the period 1960-2000

the average attainment in Korea rose 7.23 years while in Uruguay it rose only 2.22 years. As pointed out by Duryea and Pages (2002) for the Latin American case: "In general, the prospects are dim because progress in raising average schooling levels has been slow even under the best historical scenarios."

So some questions arise: What happened? How bad this gap could be? Is there anything that can be done to reverse this situation? These are key questions for many developing countries. As poor countries are lagging behind the global pace of accumulation of skills, the unsatisfactory performance of the education sector seems to be a central aspect of the problem, so, there would be broad scope for policy action to reverse the situation. Indeed education policy can regulate the level of speed of the accumulation process, by means of suitable strategies aiming at improving systemic efficiency.

Let focus on the Korea-Uruguay case. For computation purposes, efficiency units are disregarded. Let assume that the central authority set as an objective to achieve a certain objective level of skill to unskilled labor ξ^* in a minimum period of time, given the budget constraint. Assuming a constant variation in each magnitude it is possible to describe the route to the target as:

$$\frac{L_{S0} + T dL_S}{L_{U0} + T dL_U} = \left(\frac{L_S}{L_U}\right)^*$$

where T is the time required to reach a desired target of skilled to unskilled ratio $(L_S/L_U)^*$, from a starting situation L_{S0}/L_{U0}, given the flows dL_S, dL_U. By extension, the initial and the target ratios can be written as: $\xi^* = (L_S/L_U)^*$ and $\xi_0 = L_{S0}/L_{U0}$, then the following expression for results:

$$T = \frac{\xi^* - \xi_0}{\xi - \xi^*}(L_{U0}/dL_U)$$

where it holds that $\partial T/\partial \theta > 0$ and $\partial T/\partial K < 0$. Assuming $\xi^* - \xi_0 > 0$, the above expression shows that time will be higher the farther is the target from the current situation, and it will be lower the greater is the marginal ratio. But the target is only feasible when $\xi > \xi^*$, so speed is not the only problem but feasibility too.

For instance, in 2006 $\xi = 0.16$ in Uruguay and $\xi_0 = 0.11$ (see Doneschi and Patron, 2010), while in Korea $\xi_0 = 0.46$ (UNESCO data). An Korea-like ratio as target ($\xi^* = 0.46$) is unfeasible for Uruguay given 2006 values, as $\xi < \xi^*$. So the speed may not be the problem, as it may be is much worse: the target is unfeasible if the generation of skills follow the current trend as feasibility requires $\xi > 0.46$. This value would imply that the share of tertiary graduate in the flow should rise to at least 32% (from 14% in 2006) to be able to reach Korea current levels of skills (i.e. 2005/6) at some point in time (soon or later).

So, is catching up the developed world levels of skills always possible for developing countries? The poor performance of the education sector in many countries cast serious doubts on this, though education policy could regulate the level and speed of the accumulation process by means of suitable strategies aiming at improving systemic efficiency. So, the question that not only the speed of skill accumulation that matter but that the question might be one of unfeasibility.

7. Conclusions

The ratio of skill to unskilled labour stocks in the economy is widely acknowledged to have an important role for development. Can education policy affect the evolution of this ratio? This paper shows that it can, and it also shows that the actual effect of education policy depends on the allocation rule of the budget across educational levels.

The skilled-to-unskilled ratio of the inflow of labour created depends on the internal

efficiency of the education sector. The cumulative nature of the education process leads to asymmetries between educational levels, particularly in presence of systemic inefficiency. This is so because school failure at the basic level leaves the few entrants to higher education with high output per student: few highly qualified graduates. The consideration of a stylized hierarchical education model allows us to develop analytical conditions under which the allocation rule favours the accumulation of skills.

The analysis has implication for policy makers in developing countries, where skill formation is much needed, as it shows that observed allocation rules usually violate the maximization condition by the assignment of higher than optimal resources to higher education. A further implication is that, as long as the marginal skill-to-unskilled ratio regulates the wage gap, a less-than-maximum value would worsen the wage distribution.

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APPENDIX

Preliminary results

The educational budget constraint is:

$$K = K_B + K_H$$

By definition $k = k_B + (1-\theta)k_H$, where $k = K/E_B$. Thus,

$$dk_H = -\left[1-\theta-(k-k_B)\right]dk_B/(1-\theta)^2 \qquad (A.1)$$

Also, after some manipulation, the above expression can be written as:

$$\varepsilon_{k_H k_B} = \frac{K_B}{K_H} + \frac{\theta}{1-\theta}\varepsilon_{\theta\, k_B} \qquad (A.2)$$

where $\varepsilon_{\theta\, k_B} = -\partial\theta/\partial k_B \; k_B/\theta$ and $\varepsilon_{h_H k_B} = -dk_H/dk_B \; k_B/k_H$

Demonstration 1

Totally differentiating ξ results in the following:

$$d\xi = \frac{\partial\xi}{\partial k_H}dk_H + \frac{\partial\xi}{\partial k_B}dk_B \qquad (A.3)$$

where

$$\frac{\partial\xi}{\partial k_B} = -\frac{1}{\theta q_B}\left[(1-\theta)\frac{q_H}{q_B}q_B' + (q_B+q_H)\frac{\theta'}{\theta}\right] \qquad (A.4)$$

and

$$\frac{\partial\xi}{\partial k_h} = \frac{1-\theta}{\theta}\frac{q_H'}{q_B} \qquad (A.5)$$

While the sign of $\partial\xi/\partial k_H > 0$, that for $\partial\xi/\partial k_B$ is indeterminate. The conditions under which $\partial\xi/\partial k_B > 0$ are easy to find. Considering the following definitions, $S_H = q_H/f_H$ and $\varepsilon_{\theta\, q_B} = -\partial\theta/\partial q_B \; q_B/\theta = -\theta' q_B/\theta$, and substituting in Equation A.3 after some manipulation, the following is obtained

$$\frac{\partial\xi}{\partial k_B} > 0 \Leftrightarrow \varepsilon_{\theta q_B} > S_H(1-\theta).$$

Demonstration 2

Inserting in A.1, A.4, and A.5 in A.3, using A.2 and the definitions $\varepsilon_{\theta\, k_B} = -\partial\theta/\partial k_B\ k_B/\theta$ and $\varepsilon_{\theta\, k_H} = -\partial\theta/\partial k_H\ k_H/\theta$, after some manipulation, the result is as follows

$$\frac{d\xi}{dk_B} > 0 \Leftrightarrow \frac{\varepsilon_{\theta\, k_B}}{\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}} > S_H(1-\theta)$$

.

It also can be shown that

$$\varepsilon_{\xi\, k_B} = \frac{\varepsilon_{\theta\, k_B}}{1-\theta} - S_H\left[\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}\right]$$

Inserting A.2 in the above expression, and defining $B_H = K_H/K$, and considering the "quasi-neutral" assumption that $\varepsilon_{q_B k_B} = \varepsilon_{q_H k_H} = \varepsilon_{q_j k_j}$, the following is obtained:

$$\varepsilon_{\xi\, k_B} = \frac{\varepsilon_{\theta\, k_B}}{1-\theta}\left[1 - S_H\varepsilon_{q_j k_j}\theta\right] - \frac{S_H}{B_H}\varepsilon_{q_j k_j}$$

.