## *2. Publicly finance schools productivity comparison for Basque Country through a new educational Malmquist index approach*

Eva Crespo Cebada, *Universidad de Extremadura*
Francisco Pedraja Chaparro, *Universidad de Extremadura*
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**Abstract**
It is well known in the educational literature that public and government dependent private schools show different production technologies and present differences in the productivity of their educational inputs due to management issues. On the one hand the students attending public schools differ in their socioeconomic characteristics from students in the government dependent private (private management and funded by the government) ones. In this paper we propose a new non-parametric Educational Malmquist index approach in order to analyze total factor productivity changes and divergences between publicly financed schools when only a pseudo-panel database is available. This paper includes an empirical analysis for the Spanish Region Basque Country, who has a representative sample in the Programme for International Student Assessment (PISA) 2003, 2006 and 2009. PISA includes a wide variety of background information on the students collected by student questionnaires and about schools resources. The results suggest a higher annual productivity change for government dependent private schools, mostly due to technical divergences among both school types. Similarly, the general productivity evolution among, public and government dependent private, schools within 2003-2009 benefits government dependent private schools in all cases, although this figures seem not to be significant.

Keywords: Efficiency, Productivity, Education, PISA, Malmquist.

## 1. Introduction

One of the main goals in the field of economics of education is to analyze the inefficiency behaviours in the learning process. The sources of inefficiency may be due to multiple reasons such as the way in which resources are organized and managed, the motivation of the agents involved in the process or the structure itself of the educational system (Nechyva, 2000; Woessman, 2001).

The recently increase of national and international programs to evaluate the scholar achievement during last decades shows the higher policy concern about educational performance. Hence, last years some international projects have been developed in order to evaluate the educational achievements in which are considered the vehicular disciplines: Science, Mathematics and Lecture. The most important international programs are TIMSS *(Third International Mathematics and Science Study)*, PISA *(Programme for International Student Assessment)* and PIRLS *(Progress in International Reading Literacy Study)* although many countries perform their own evaluations e.g. the National Assessment of Educational Progress (NAEP) in the United States.

The main advantage of these programs is that provide an external evaluation of educational results with the aim of identifying causes of school failure allowing to policy makers and school principals to go into their management strengths and weakness in depth. However, the comparison of students or schools behaviours along the time using these international studies is not possible due to participant schools and students differ from one wave to another.

In order to tackle the inefficiency measurement issue in education many studies have used non parametric Data Envelopment Analysis (Bessent and Bessent, 1980; Charnes, Cooper, and Rhodes, 1981 and Bessent et al., 1982; Cordero et al., 2010a[77]) and other parametric methodologies (Christensen, Jorgenson, and Lau, 1971; Gyimah-Brempong and Gyapong, 1992; Deller and Rudnicki, 1993, Grosskopf et al., 1997, Perelman and Santín, 2008, Cordero et al., 2010b).

In this paper we propose the use of the well known Malmquist Index in order to obtain a measurement of productivity divergences between Public Schools (PS) and Government Dependent Private Schools (GDPS) within three time periods (2003-2009). With this aim we provide an empirical application to Basque Country educational data from the *Programme for International Student Assessment* (PISA), implemented in 2003, 2006 and 2009 by the Organization for Economic Cooperation and Development (OECD). PISA includes a wide variety of background information on the students collected by student questionnaires and about schools resources, (for an extensive review see OECD, 2007a, 2007b and 2009).

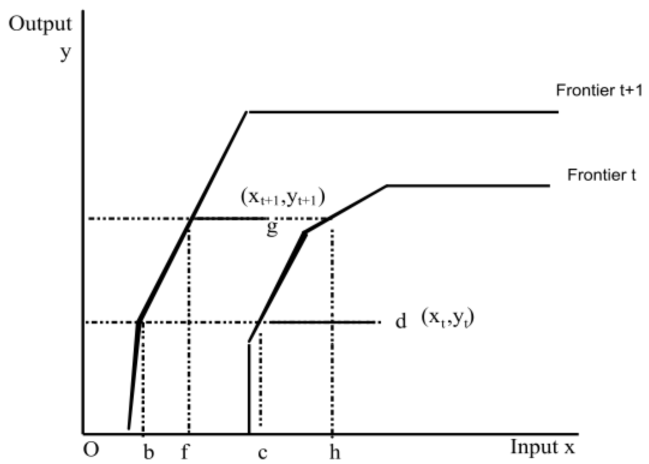
The paper is organized as follows. Section 2 provides an overview about the Malmquist Index together with our estimation strategy. In Section 3 data set and selected inputs and outputs are described. Section 4 provides results and a discussion of our empirical analysis and the final section offers some conclusions and future lines for research.

## 2. Methodology

Malmquist Index (MI) was proposed by Caves, Christensen and Diewet (1982) with the aim of measuring the productivity changes within two time periods as the distance between a decision making unit (DMU) and the frontier for each period. The index is built using different Data Envelopment Analysis (DEA) programs, so no assumptions, beyond monotonicity and convexity, about the production technology are required. Hence, it is especially attractive in the educational context, where multiple inputs and output are involved and prices are unknown or difficult to estimate.

The MI provides a measure of the total productivity factors *(TPF)* evolution and their components along the time, so the *TPF* is explained by the efficiency change, which is known as catch-up effect, and the technology change (frontier shift). Figure 1 illustrates the *TFP* change for a *DMU* in two periods.

**Figure 1.** Productivity change in one input and one output set




77 For an empirical survey of frontier efficiency techniques in education, see Worthington (2001).

In Figure 1, *DMU* d (g) employs in period t (t+1) $x_t$ ($x_{t+1}$) units of input to produce $y_t$ ($y_{t+1}$) units of outputs. Through these quantities the MI measures the catch-up effect as the ratio $\frac{ef}{eg}\Big/\frac{ac}{ad}$. When the index is greater (less) than one the *DMU* improves (make worse) its efficiency. The frontier or technology shift is denoted by $\sqrt{\frac{ac}{ab}\cdot\frac{bh}{bf}}$, where values greater (less) than one implies a production frontier upward (downward) movement.

To formalize the index we first assume constant returns to scale (CRS). Defining a vector of inputs $x = (x_1, \ldots, x_K) \in \Re^{K+}$ and a vector of outputs $y = (y_1, \ldots, y_M) \in \Re^{M+}$, a feasible multi-input multi-output production technology for a period of time t (t = 1….,T) can be defined using the output possibility set $P^t(x^t)$. This output possibility set can be produced using the input vector $x^t$: $P^t(x^t) = \{y^t: x^t \text{ can produce } y^t\}$, which is assumed to satisfy the set of axioms described in Färe and Primont (1995). This technology can be also defined as the output stochastic distance function proposed by Shephard (1970):

$$D^t(x^t, y^t) = \inf\{\theta : \theta > 0, (x^t, y^t/\theta) \in P^t(x^t)\} \tag{1}$$

From Equation (1), if $D^t(x^t, y^t) \leq 1$ then $(x^t, y^t)$ belongs to the production set $P^t(x^t)$ and, additionally, when $D^t(x^t, y^t) = 1$ $y^t$ is located on the outer boundary of the output possibility set.

Following Färe et al. (1994) we may define the Malmquist productivity index from the distance function, $D^t$, and the inputs - outputs endowments $x^t$, $y^t$ for each period of time t (t = 1….,T). The analytical expression of the index would be:

$$M(x^{t+1}, y^{t+1}, x^t, y^t) = \frac{D^{t+1}(x^{t+1}, y^{t+1})}{D^t(x^t, y^t)} * \left[\left(\frac{D^t(x^{t+1}, y^{t+1})}{D^{t+1}(x^{t+1}, y^{t+1})}\right)\left(\frac{D^t(x^t, y^t)}{D^{t+1}(x^t, y^t)}\right)\right]^{1/2} = TEC * TC \tag{2}$$

where a higher than one index implies productivity improvements and lower than one productivity losses[78].

Equation (2) may be decomposed into two items. The first one, the technical efficiency change (TEC) catches improvements (reductions) on efficiency in period t+1 when TEC > 1 (TEC < 1), whereas *TEC* = 1 indicates no changes on technical efficiency. The second measure represents the technological change *(TC)* in period t+1, whose sign may be analysed in a similar way than *TEC* although both measures may have different directions.

Furthermore the index may be calculated assuming constant returns to scale *(CRS)* and variable returns to scale *(VRS)*, which allow us to decompose the efficiency change into pure efficiency and scale efficiency changes as shows the following expression:

$$TEC = \frac{D_o^{t+1}(X^{t+1}, Y^{t+1})}{D_o^t(X^t, Y^t)} = \frac{D_{VRS}^{t+1}(X^{t+1}, Y^{t+1})}{D_{VRS}^t(X^t, Y^t)} \frac{\dfrac{D_{CRS}^{t+1}(X^{t+1}, Y^{t+1})}{D_{VRS}^{t+1}(X^{++1}, Y^{t+1})}}{\dfrac{D_{CRS}^t(X^t, Y^t)}{D_{VRS}^t(X^t, Y^t)}} = PEC * SEC \tag{3}$$

78 This productivity index is the geometric mean of two productivity index, where the first one takes t period as reference and the second one t+1, avoiding arbitrary selection in the period of reference.

where the first item catches the pure efficiency change *(PEC)* with respect to variable returns to scale frontier and the second one represents the scale efficiency change *(SEC)*, which reflects changes between both *CRS* and *VRS* frontiers.

Therefore, following Ray and Desli (1997), the Malmquist Index considering *VRS*, comprises three elements: the pure efficiency change *(PEC)*, the scale efficiency change *(SEC)* and the technology change *(TC)*.

$$M\left(x^{t+1},y^{t+1},x^{t},y^{t}\right)=\frac{D^{t+1}(x^{t+1},y^{t+1})}{D^{t}(x^{t},y^{t})}*\left[\left(\frac{D^{t}(x^{t+1},y^{t+1})}{D^{t+1}(x^{t+1},y^{t+1})}\right)\left(\frac{D^{t}(x^{t},y^{t})}{D^{t+1}(x^{t},y^{t})}\right)\right]^{1/2}=TEC*TC \quad (4)$$

The purpose of our study is to analyse productivity differences among publicly financed schools using the MI approach. Thus, we propose a non-parametric Educational Malmquist (EM) in order to obtain productivity divergences between PS and GDPS in the same year instead of making the comparison across the time. With the aim to perform this analysis we need a pseudo-panel database as PISA or TIMSS, where sampled schools are representative from publicly financed educational systems.

Equation (5) shows the expression for this EM, where *"C"* and *"P"* super-indexes indicate GDPS and PS respectively, which substitute the super-indexes *t* and *t+1*, and subindex *i* indicates the time period:

$$EM_i=\frac{D^{C}_{VRS}(X^{C},Y^{C})}{D^{P}_{VRS}(X^{P},Y^{P})}\,\frac{\dfrac{D^{C}_{CRS}(X^{C},Y^{C})}{D^{C}_{VRS}(X^{C},Y^{C})}}{\dfrac{D^{P}_{CRS}(X^{P},Y^{P})}{D^{P}_{VRS}(X^{P},Y^{P})}}\cdot\left[\frac{D^{C}(x^{P},y^{P})}{D^{P}(x^{P},y^{P})}x\frac{D^{C}(x^{C},y^{C})}{D^{P}(x^{C},y^{C})}\right]^{1/2} \quad (5)$$

The expression above shows average productivity differences in one year between both PS and GDPS. The EM index may be decomposed into three components: the first one refers to pure efficiency differences among them, which are the differences in distances for each management system to its own *VRS* frontier, the second item is the scale efficiency divergence, that indicates how separated are both *CRS* and *VRS* frontiers for PS and GDPS respectively, and the last one is the technology difference between both school types. This approach allows us to compare mean productivities between both educational systems with the aim of analysing efficiency and technology differences among both PS and GDPS. Hence, a higher (smaller) than one index implies that GDPS are more (less) productive than PS. Therefore the EM index expressed as the product of these three components would be:

$$EMI=\frac{(Oe/Oq)}{(Of/Op)}\cdot\left[\frac{\dfrac{(Oc/Oq)}{(Oe/Oq)}}{\dfrac{(Ob/Op)}{(Of/Op)}}\right]\cdot\sqrt{\left[\frac{(Og/Oq)}{(Oc/Oq)}\cdot\frac{(Ob/Op)}{(Oa/Op)}\right]} \quad (6)$$

As it was mentioned above, divergences in productivity may be explained by differences among both school types in three components: the pure efficiency difference *(PED)*, the scale efficiency difference *(SED)* and the technological difference *(TC)*. The *PED*, which indicates the distance from a GDPS to its VRS frontier respecting to the distance from a PS to its *VRS* frontier, is given by the ratio:

$$PED=\frac{(Oe/Oq)}{(Of/Op)} \quad (7)$$

The *SED* shows how separated are both *CRS* and *VRS* for a GDPS with respect to the same distance for a PS, being the expression as follows:

$$SED = \frac{\dfrac{(Oc/Oq)}{(Oe/Oq)}}{\dfrac{(Ob/Op)}{(Of/Op)}} \qquad (8)$$

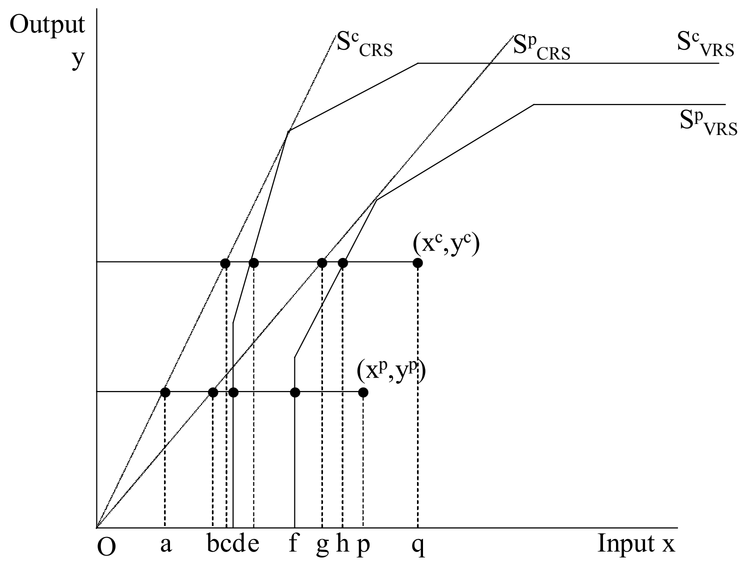
Finally, TD among schools referring to CRS frontiers, are showed by:

$$TD = \sqrt{\left[\frac{(Og/Oq)}{(Oc/Oq)} \cdot \frac{(Ob/Op)}{(Oa/Op)}\right]}\text{[79]} \qquad (9)$$

Figure 2 illustrates these concepts in a simple one output-one input setting in one period. Let assume that the frontier $S^i_{VRS}$ represents variable returns to scale *(VRS)* technology and the constant returns to scale *(CRS)* technology is indicated by the line $S^i_{CRS}$, where the super-index indicates the school ownership, PS *(P)* and GDPS *(C)*.

Moreover the average inefficient public (government dependent private) school consumes $x^P$ ($x^C$ ) input and produces $y^P$, ($y^C$) output.

**Figure 2.** Productivity divergences between public and government dependent private schools




Furthermore, we are interesting in analysing productivity changes within two periods using the ratio of two EM expressions. Then the productivity deviation of GDPS with respect to PS within t and t+1 is as follows:

$$EMC = \frac{M_t}{M_{t+1}} \qquad (10)$$

Equation (10) indicates productivity gains for PS when *EMC* > 1 or productivity losses when *EMC* < 1. Similarly, the pure efficiency change *(PEC)*, the scale efficiency change *(SEC)* and the technology changes *(TC)* between PS and GDPS within the two periods may be explained by the following ratios:

$$PEC = \frac{PED_t}{PED_{t+1}} \quad TC = \frac{TD_t}{TD_{t+1}} \quad SEC = \frac{SED_t}{SED_{t+1}} \qquad (11)$$

79 In this simple graphical example the two components of TD are identical, but this will not in general be the case.

where a higher than one ratio implies PS are more efficiency and/or advanced technologically than GDPS are.

For empirical purpose we propose four alternatives in order to match both PS and GDPS samples when the size of these samples is different, $m$ and $n$ respectively.

These alternatives allow us to obtain a robust matching of the units, taking into account that results must be analysed in the mean, so unit are different across samples. Thus, our proposal consists of matching different units instead of the same unit in different time periods, so our analysis is only valid at the mean value.

The Alternative 1 consists of matching samples randomly. Let assume that $n > m$. Therefore, we select $m$ units from the largest sample $n$, being the remainder units using to build the productive frontier but they are not considered to obtain the Educational Malmquist index.

The Alternative 2 consists of removing a number of units from the smaller sample to obtain the same number of units than in the largest one. Let assume that $n > m$ then:

- If $m*2 > n$; $n - m$ units are removed randomly from the m sample to equal n.
- If $m*f > n$; where $f$ is a natural number equals or higher than 2 $(f= 2, 3,\ldots, F)$ and it is used to match the m sample multiplied by $f$.
- If $m*2 < n$; where $f$ is a natural number equals or higher than 2 $(f= 2, 3,\ldots, F)$ and it is used to match the m sample multiplied by $f$. Then, $n – m*f$ units are removed randomly from the original $m$ sample to equal $n$.

Alternative 3 consists of the following steps:

- A Data Envelopment Analysis *(DEA)* is estimated for the higher sample.
- Let assume that $n > m$. Then, $m$ units are selected from the higher sample, $n$, maintaining the percentage of efficient units.
- Unmatched units are used to build the productive frontier and to evaluate the other units, although not to obtain the mean indexes.

Alternative 4 consists of the following steps:

- A *DEA* is estimated for the smaller sample.
- Let assume that $n > m$. Then, $m$ units are randomly selected from the smaller sample, $m$, to achieve $n$ maintaining the percentage of efficient units.

- *If* $m*2 > n$; $n - m$ units are removed randomly from the $m$ sample to equal $n$.
- *If* $m*f > n$; where $f$ is a natural number equals or higher than 2 $(f= 2, 3,\ldots, F)$ and it is used to match the m sample multiplied by $f$.
- *If* $m*2 < n$; where $f$ is a natural number equals or higher than 2 $(f= 2, 3,\ldots, F)$ and it is used to match the $m$ sample multiplied by $f$. Then, $n – m*f$ units are removed randomly from the original $m$ sample to equal $n$.

Finally, unmatched units are used to build the productive frontier and to evaluate the other units, although not to obtain the mean indexes.

Finally, in order to obtain statistical test of results, we use the bootstrap methodology, proposed by Simar and Wilson, 1999, with the aim of calculating confidence intervals for the

Educational Malmquist and its components. This methodology consists of estimating the empirical distribution of non-parametric measure, which allows us to determine the robustness of efficiency and productivity analysis results.

## 3. Dataset and variables

In our empirical analysis, we use Basque Country school data from *PISA* 2003, 2006 and 2009, which provide us with data from 15 years old students. The methodology described in section 2 is carried out for each region separately, so Spanish regions are actually fully responsible for the management of educational resources since 2000.

One of the main advantages of the PISA study is that it does not evaluate cognitive abilities or skills through using one single score but each student receives a score in each test within a continuous scale. In this way, PISA attempts to collect the effect of particular external conditioning factors not depending on the students when taking the test, namely being ill or becoming very nervous, among other random factors. Furthermore, it also involves that measurement error in education is not independent from the position of the student in the distribution of results. Precisely, students with very low or high results have higher associated measurement errors and higher asymmetry in error distribution.

Likewise, PISA also collects an extensive dataset on these variables through two questionnaires: one completed by the students themselves and another one filled out by principals. From these data, it is possible to extract a great amount of information referred to the main determining factors of educational performance represented by variables associated to familiar and educational environments as well as to school management and educational supply.

### *Outputs and plausible values*

The true output as result of an individual education is very difficult to measure empirically due to its inherent intangibility. Education does not only consist of the ability of repeating information and answering questions, but it also involves the skills to interpret the information and learn how to behave in the society. Unfortunately, it is really hard to measure all of them. But perhaps, according to Hoxby (2000), the most important reason could be that both policy makers and parents use this criterion to evaluate the educational output and its subsequent information to choose the school for their children and even their place of residence.

In this study we use the results obtained by students in the three competences evaluated in PISA (mathematics, reading comprehension and sciences) as school output. As it has already been mentioned, the study uses the concept of plausible values to measure the performance of students, since measures in these subjects have a wide margin of error due to the fact that the measuring concept is abstract and they are subject to the special circumstances of students and the special circumstances on the date of the exam. These values are random values obtained from the distribution function of results estimated from the answers in each test. They can be interpreted as a representation of the ability range for each student[80] (Wu and Adams, 2002).

Plausible values in the three tests are used as outputs in the efficiency analysis. In order to obtain correct results and avoid problems of bias in estimations it will be necessary to calculate five different EM index for each trio of plausible values and take the mean value afterwards, instead of using mean values to obtain one EM index (*OCDE*, 2005).

80 For a review of plausible values literature see Mislevy et al. (1992). For a concrete Studio of Rasch model and how obtain feasible values in PISA, see OECD (2005).

### *Inputs*

In order to calculate the EM index we have used four different inputs that are directly involved with learning (*PARED, HISEI, MATEDU* and *STRATIO*). *PARED* is the index of highest level of parental education in number of years of education according to the International Standard Classification of Education (*ISCED, OECD*, 1999) and *HISEI* is the index of highest parental occupation status according to International Socio-economic index of Occupational Status (*ISEI*, Ganzeboom et al., 1992). *MATEDU* is an index of quality of scholar resources derived from school manager's responses. All questionnaires contain several items related with the school deficiencies on those issues, but some items are different across the three waves. So ten coincident items were selected in each sample and the school receive one point in case the manager's response would be there is not deficiency in each item[81]. The maximum (minimum) punctuation for each school is ten (zero) points, which indicates an excellent (dreadful) educational input[82]. Finally, *STRATIO* is a ratio between total number of students in school and total number of teachers weighted on their dedication (part-time teachers contributes 0.5 and full-time teachers 1).

Table 1 shows the mean value for the three outputs – students' results on Mathematics, Readings and Science- and for the inputs commented above – been two of them related with the socioeconomic background of the students and the other two inputs refer to the scholar resources. The figures below indicate that the students' results are higher for GDPS in all disciplines. However, the mean socioeconomic background, measured by both variables *PARED* and *HISEI* is lower in PS. Similarly, GDPS present a higher quality of resources, *MATEDU*, although the student-teacher ratio *(STRATIO)* benefits to PS, were the ratio is higher than for the other ones, which implies that each teacher is in charge of a more reduced group of students.

**Table 1.** Descriptive statistics of outputs and inputs in Basque Country

| | *2003* | | | | *2006* | | | | *2009* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Measures* | *Mean* | *Std. Dev.* | *Min.* | *Max.* | *Mean* | *Std. Dev.* | *Min.* | *Max.* | *Mean* | *Std. Dev.* | *Min.* | *Max.* |
| *Variables* | *Public Schools* | | | | | | | | | | | |
| *Math* | 488.67 | 30.31 | 390.79 | 550.37 | 477.99 | 53.47 | 367.98 | 574.68 | 497.61 | 40.53 | 389.11 | 569.44 |
| *Read* | 480.69 | 30.40 | 396.83 | 541.46 | 464.82 | 54.95 | 337.43 | 584.09 | 480.02 | 34.07 | 405.15 | 529.89 |
| *Science* | 467.73 | 32.50 | 372.59 | 534.77 | 474.26 | 44.51 | 366.33 | 562.39 | 481.58 | 28.73 | 414.26 | 543.18 |
| *Pared* | 11.61 | 1.22 | 8.50 | 13.97 | 11.86 | 1.71 | 8.25 | 15.68 | 12.96 | 1.30 | 9.82 | 15.50 |
| *Hisei* | 42.37 | 6.79 | 31.13 | 60.23 | 44.47 | 6.78 | 32.33 | 62.70 | 45.55 | 6.06 | 32.30 | 59.53 |
| *Matedu* | 6.21 | 3.54 | 1.00 | 11.00 | 8.48 | 2.51 | 1.00 | 11.00 | 9.01 | 1.55 | 6.00 | 11.00 |
| *Stratio* | 14.62 | 2.45 | 9.41 | 20.88 | 15.54 | 2.89 | 11.30 | 24.30 | 15.02 | 2.99 | 9.15 | 23.71 |
| *Obsv.* | 53 | | | | 56 | | | | 68 | | | |
| *Variables* | *Government-Dependent Private Schools* | | | | | | | | | | | |
| *Math* | 509.39 | 34.68 | 426.83 | 581.83 | 512.45 | 38.62 | 402.79 | 584.17 | 520.73 | 34.77 | 436.98 | 593.48 |
| *Read* | 509.16 | 37.44 | 424.02 | 592.41 | 500.62 | 39.03 | 395.85 | 590.51 | 509.52 | 36.64 | 411.43 | 580.10 |
| *Science* | 495.02 | 36.69 | 411.30 | 564.49 | 505.40 | 36.91 | 426.95 | 597.50 | 507.56 | 35.15 | 391.50 | 576.13 |
| *Pared* | 12.28 | 1.21 | 8.50 | 14.65 | 12.89 | 1.57 | 9.09 | 16.17 | 13.80 | 1.37 | 10.41 | 16.25 |
| *Hisei* | 46.78 | 8.64 | 33.62 | 67.88 | 49.33 | 8.42 | 34.62 | 68.95 | 51.53 | 8.07 | 35.61 | 71.57 |
| *Matedu* | 7.89 | 3.19 | 1.00 | 11.00 | 8.06 | 2.73 | 1.00 | 11.00 | 9.08 | 1.85 | 4.00 | 11.00 |
| *Stratio* | 6.78 | 1.18 | 4.23 | 10.31 | 6.89 | 1.25 | 4.13 | 10.37 | 7.41 | 1.72 | 3.96 | 15.66 |
| *Obsv.* | 70 | | | | 81 | | | | 90 | | | |

81 The selected item are: 'Qualified teachers on Science', 'Qualified teachers on Mathematics', 'Qualified teachers on Lecture', 'Any other Personal Support', 'Science laboratory equipment', 'Educational material', 'Computers', 'Software', 'Library resources', 'Audiovisual resources'.

82 This variable has been rescaled in order to avoid zeros in the empirical analysis.

## 4. Results

This section presents the main results obtained in our analysis for Basque Country. Our methodology allows us comparing *PS* and *GDPS* productivity changes between 2003-2006 and 2006-2009.

Tables 2-3 report the results after applying this EM methodology for Basque Country in 2003, 2006 and 2009 using alternatives 1-4 for matching. Thus, the first column of each year of Table 4 shows the EM index, where PS are considered as period t and GDPS as period t+1. After this measure, we report all its components: the Efficiency Divergence (*ED*), the Technological Difference (*TD*) and finally the Pure Efficiency Difference (*PED*) and the Scale Efficiency Difference (*SED*) respectively. Similarly, Table 3 represents the productivity gains between both *PS* and *GDPS* within 2003-2009 using all alternatives. The first column for each year of Table 3 shows the EMC within two time periods (*EMC*) and its components: the Efficiency Change (*EC*), the Technological Change (*TC*) and finally the Pure Efficiency Change (*PEC*) and the Scale Efficiency Change (*SEC*) respectively. Finally, Tables 6-7 indicate the 90%, 95% and 99% confidence intervals (Simar y Wilson, 1999) for the EM and EMC, respectively, being *LB* the Lower Band and *UB* the Upper Band.

This confidence interval allows us to obtain statistical test of the different measures.

Several conclusions may remove from Table 2. Firstly, *GDPS* are generally more productive than *PS* independently of the alternative used for matching samples. However the productivity difference among both school types is reduced year by year, going from 2% using alternative 1 to 4% using alternative 2 between 2003 and 2009. On the other hand, it is observed a peak in this tendency independently of the alternative we use to match the different samples. Thus, GDPS are even more productive related to PS in 2006, although as Table 4 shows these results are no significant for any alternative. Moreover, productivity divergences are mostly explained by the technological gap among PS and GDPS, but again the figures are not significant for any alternative. Finally, the simulation for Basque Country seems to indicate that the results are not sensitive to the matching alternative, so there no relevant discrepancies in the educational Malmquist and its components for the different alternatives used to match the school samples.

Similarly, results from Table 3 indicates that PS are significantly more productive than *GDPS* within 2003-2009, although this apparently advantage of the public system reflects the reduction in the productivity difference among both school ownership from 2003 to 2009. On the other hand, the technology gap is the main component in the EM, especially for alternative 2 and 4, which are significant at 99%. Nevertheless, the efficiency component jointly with the technology one explain the productivity change within 2003-2009 using alternatives 1 and 3, in fact PS present a higher and significant pure efficiency than GDPS. Then, although GDPS are more advantage technologically, PS are reducing the distance to its frontier, so they improve their efficiency level. Finally, the apparently superiority of *GDPS* in Basque Country may be explained because the differences between PS and *GDPS* are reducing year by year, although PS are more productive for an individual year, the productivity evolution benefits for GDPS.

**Table 2.** Educational Malmquist Index in Basque Country

| | | *2003* | | | | | *2006* | | | | | *2009* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Region* | *Statistic* | *EM* | *ED* | *TD* | *PED* | *SED* | *EM* | *ED* | *TD* | *PED* | *SED* | *EM* | *ED* | *TD* | *PED* | *SED* |
| *A1* | *Mean* | 1.2405 | 0.9934 | 1.2522 | 1.0061 | 0.9873 | 1.3664 | 0.9688 | 1.4189 | 0.9888 | 0.9797 | 1.2240 | 0.9864 | 1.2452 | 0.9964 | 0.9900 |
| | *Std.Dev* | 0.0195 | 0.0109 | 0.0227 | 0.0080 | 0.0067 | 0.0254 | 0.0151 | 0.0270 | 0.0094 | 0.0102 | 0.0307 | 0.0111 | 0.0362 | 0.0066 | 0.0070 |
| A2 | *Mean* | 1.2610 | 1.0021 | 1.2638 | 1.0165 | 0.9858 | 1.3627 | 0.9842 | 1.3922 | 1.0046 | 0.9797 | 1.2229 | 0.9952 | 1.2336 | 1.0029 | 0.9924 |
| | *Std.Dev* | 0.0232 | 0.0101 | 0.0238 | 0.0067 | 0.0064 | 0.0230 | 0.0132 | 0.0229 | 0.0078 | 0.0088 | 0.0277 | 0.0103 | 0.0320 | 0.0055 | 0.0067 |
| A3 | *Mean* | 1.2516 | 0.9947 | 1.2620 | 1.0062 | 0.9885 | 1.3703 | 0.9732 | 1.4176 | 0.9908 | 0.9822 | 1.2253 | 0.9887 | 1.2441 | 0.9986 | 0.9901 |
| | *Std.Dev* | 0.0201 | 0.0110 | 0.0231 | 0.0081 | 0.0067 | 0.0266 | 0.0145 | 0.0267 | 0.0094 | 0.0095 | 0.0303 | 0.0109 | 0.0347 | 0.0065 | 0.0070 |
| A4 | *Mean* | 1.2379 | 0.9961 | 1.2452 | 1.0137 | 0.9827 | 1.3631 | 0.9809 | 1.3999 | 1.0049 | 0.9761 | 1.2068 | 0.9940 | 1.2185 | 1.0018 | 0.9922 |
| | *Std.Dev* | 0.0121 | 0.0091 | 0.0141 | 0.0063 | 0.0059 | 0.0263 | 0.0131 | 0.0219 | 0.0077 | 0.0087 | 0.0179 | 0.0099 | 0.0207 | 0.0055 | 0.0065 |

**Table 3.** Productivity gains between *PS* and *GDPS* within 2003-2009 in Basque Country

| | | *Ratio 03-06* | | | | | *Ratio 06-09* | | | | | *Ratio 03-09* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Region* | *Statistic* | *CM* | *CE* | *CT* | *CEP* | *CEE* | *CM* | *CE* | *CT* | *CEP* | *CEE* | *CM* | *CE* | *CT* | *CEP* | *CEE* |
| *A1* | *Mean* | 0.9081 | 1.0256 | 0.8828 | 1.0176 | 1.0079 | 1.1169 | 0.9822 | 1.1404 | 0.9924 | 0.9897 | 1.0069 | 1.0036 | 1.0031 | 1.0049 | 0.9987 |
| | *Std.Dev* | 0.0192 | 0.0193 | 0.0235 | 0.0126 | 0.0126 | 0.0300 | 0.0187 | 0.0386 | 0.0115 | 0.0123 | 0.0142 | 0.0078 | 0.0170 | 0.0051 | 0.0049 |
| A2 | *Mean* | 0.9255 | 1.0183 | 0.9080 | 1.0119 | 1.0063 | 1.1148 | 0.9890 | 1.1293 | 1.0017 | 0.9873 | 1.0156 | 1.0034 | 1.0124 | 1.0068 | 0.9967 |
| | *Std.Dev* | 0.0177 | 0.0172 | 0.0221 | 0.0102 | 0.0113 | 0.0267 | 0.0168 | 0.0347 | 0.0095 | 0.0112 | 0.0133 | 0.0071 | 0.0162 | 0.0043 | 0.0046 |
| A3 | *Mean* | 0.9136 | 1.0223 | 0.8905 | 1.0157 | 1.0065 | 1.1188 | 0.9844 | 1.1404 | 0.9922 | 0.9921 | 1.0108 | 1.0030 | 1.0074 | 1.0038 | 0.9992 |
| | *Std.Dev* | 0.0197 | 0.0188 | 0.0230 | 0.0125 | 0.0118 | 0.0300 | 0.0184 | 0.0382 | 0.0116 | 0.0120 | 0.0139 | 0.0077 | 0.0164 | 0.0051 | 0.0049 |
| A4 | *Mean* | 0.9084 | 1.0157 | 0.8897 | 1.0088 | 1.0068 | 1.1296 | 0.9869 | 1.1492 | 1.0031 | 0.9839 | 1.0129 | 1.0011 | 1.0110 | 1.0059 | 0.9952 |
| | *Std.Dev* | 0.0166 | 0.0164 | 0.0173 | 0.0099 | 0.0109 | 0.0203 | 0.0162 | 0.0261 | 0.0093 | 0.0107 | 0.0080 | 0.0068 | 0.0104 | 0.0042 | 0.0044 |

**Table 4.** Confidence interval for Educational Malmquist in Basque Country

| *Alternative* | Alternative 1 | | | | | | Alternative 2 | | | | | | Alternative 3 | | | | | | Alternative 4 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *CI* | *99%* | | *95%* | | *90%* | | *99%* | | *95%* | | *90%* | | *99%* | | *95%* | | *90%* | | *99%* | | *95%* | | *90%* | |
| *Measures* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* |
| *2003* | | | | | | | | | | | | | | | | | | | | | | | | |
| *EM* | 1.2193 | 1.2640 | 1.2134 | 1.2776 | 1.2045 | 1.3059 | 1.2350 | 1.2887 | 1.2239 | 1.3030 | 1.2096 | 1.3308 | 1.2295 | 1.2771 | 1.2225 | 1.2902 | 1.2134 | 1.3135 | 1.2233 | 1.2527 | 1.2184 | 1.2590 | 1.2120 | 1.2692 |
| *ED* | 0.9796 | 1.0077 | 0.9758 | 1.0116 | 0.9697 | 1.0184 | 0.9888 | 1.0151 | 0.9861 | 1.0191 | 0.9805 | 1.0264 | 0.9807 | 1.0089 | 0.9770 | 1.0133 | 0.9700 | 1.0212 | 0.9847 | 1.0082 | 0.9817 | 1.0119 | 0.9765 | 1.0175 |
| *TD* | 1.2287 | 1.2808 | 1.2231 | 1.2935 | 1.2135 | 1.3304 | 1.2387 | 1.2944 | 1.2341 | 1.3097 | 1.2233 | 1.3402 | 1.2373 | 1.2926 | 1.2325 | 1.3056 | 1.2224 | 1.3364 | 1.2285 | 1.2625 | 1.2238 | 1.2697 | 1.2155 | 1.2832 |
| *PED* | 0.9959 | 1.0162 | 0.9930 | 1.0191 | 0.9873 | 1.0247 | 1.0080 | 1.0250 | 1.0055 | 1.0273 | 1.0012 | 1.0320 | 0.9961 | 1.0165 | 0.9927 | 1.0196 | 0.9867 | 1.0242 | 1.0055 | 1.0218 | 1.0033 | 1.0239 | 0.9984 | 1.0282 |
| *SED* | 0.9791 | 0.9962 | 0.9766 | 0.9990 | 0.9727 | 1.0038 | 0.9780 | 0.9942 | 0.9760 | 0.9969 | 0.9721 | 1.0027 | 0.9801 | 0.9974 | 0.9782 | 0.9997 | 0.9744 | 1.0052 | 0.9753 | 0.9905 | 0.9735 | 0.9929 | 0.9705 | 0.9975 |
| *2006* | | | | | | | | | | | | | | | | | | | | | | | | |
| *EM* | 1.3296 | 1.3964 | 1.3156 | 1.4052 | 1.3004 | 1.4203 | 1.3279 | 1.3898 | 1.3149 | 1.3948 | 1.3023 | 1.4069 | 1.3308 | 1.3998 | 1.3151 | 1.4093 | 1.3015 | 1.4240 | 1.3146 | 1.3897 | 1.3053 | 1.3960 | 1.2925 | 1.4059 |
| *ED* | 0.9492 | 0.9875 | 0.9435 | 0.9934 | 0.9353 | 1.0037 | 0.9668 | 1.0009 | 0.9624 | 1.0067 | 0.9554 | 1.0142 | 0.9544 | 0.9914 | 0.9499 | 0.9965 | 0.9410 | 1.0067 | 0.9642 | 0.9981 | 0.9597 | 1.0028 | 0.9516 | 1.0112 |
| *TD* | 1.3851 | 1.4553 | 1.3766 | 1.4651 | 1.3605 | 1.4878 | 1.3630 | 1.4219 | 1.3561 | 1.4317 | 1.3448 | 1.4482 | 1.3840 | 1.4520 | 1.3765 | 1.4641 | 1.3633 | 1.4871 | 1.3721 | 1.4286 | 1.3659 | 1.4366 | 1.3533 | 1.4507 |
| *PED* | 0.9768 | 1.0004 | 0.9729 | 1.0043 | 0.9672 | 1.0116 | 0.9946 | 1.0143 | 0.9913 | 1.0173 | 0.9856 | 1.0225 | 0.9786 | 1.0029 | 0.9749 | 1.0056 | 0.9681 | 1.0108 | 0.9950 | 1.0150 | 0.9920 | 1.0177 | 0.9870 | 1.0225 |
| *SED* | 0.9663 | 0.9927 | 0.9630 | 0.9962 | 0.9573 | 1.0031 | 0.9684 | 0.9912 | 0.9655 | 0.9939 | 0.9591 | 1.0002 | 0.9701 | 0.9946 | 0.9668 | 0.9980 | 0.9607 | 1.0054 | 0.9648 | 0.9874 | 0.9624 | 0.9909 | 0.9584 | 0.9971 |
| *2009* | | | | | | | | | | | | | | | | | | | | | | | | |
| *EM* | 1.1926 | 1.2698 | 1.1843 | 1.2850 | 1.1708 | 1.3093 | 1.1936 | 1.2618 | 1.1823 | 1.2754 | 1.1718 | 1.3064 | 1.1936 | 1.2699 | 1.1836 | 1.2850 | 1.1718 | 1.3116 | 1.1846 | 1.2283 | 1.1758 | 1.2396 | 1.1680 | 1.2594 |
| *ED* | 0.9719 | 1.0006 | 0.9678 | 1.0044 | 0.9595 | 1.0125 | 0.9821 | 1.0086 | 0.9783 | 1.0124 | 0.9709 | 1.0189 | 0.9743 | 1.0029 | 0.9707 | 1.0065 | 0.9650 | 1.0134 | 0.9813 | 1.0066 | 0.9780 | 1.0108 | 0.9721 | 1.0177 |
| *TD* | 1.2059 | 1.2985 | 1.2003 | 1.3188 | 1.1878 | 1.3468 | 1.1990 | 1.2794 | 1.1941 | 1.2951 | 1.1818 | 1.3258 | 1.2064 | 1.2949 | 1.2009 | 1.3130 | 1.1890 | 1.3422 | 1.1954 | 1.2455 | 1.1910 | 1.2583 | 1.1829 | 1.2820 |
| *PED* | 0.9878 | 1.0046 | 0.9855 | 1.0072 | 0.9807 | 1.0117 | 0.9959 | 1.0097 | 0.9936 | 1.0116 | 0.9900 | 1.0153 | 0.9904 | 1.0070 | 0.9882 | 1.0093 | 0.9832 | 1.0130 | 0.9947 | 1.0090 | 0.9927 | 1.0108 | 0.9885 | 1.0140 |
| *SED* | 0.9807 | 0.9990 | 0.9779 | 1.0013 | 0.9742 | 1.0060 | 0.9835 | 1.0014 | 0.9814 | 1.0036 | 0.9774 | 1.0078 | 0.9812 | 0.9991 | 0.9788 | 1.0012 | 0.9751 | 1.0061 | 0.9840 | 1.0007 | 0.9816 | 1.0034 | 0.9782 | 1.0064 |

**Table 5.** Confidence interval for Educational Malmquist Ratios in Basque Country

| *Alternative* | Alternative 1 | | | | | | Alternative 2 | | | | | | Alternative 3 | | | | | | Alternative 4 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *CI* | *99%* | | *95%* | | *90%* | | *99%* | | *95%* | | *90%* | | *99%* | | *95%* | | *90%* | | *99%* | | *95%* | | *90%* | |
| *Measures* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* | *LB* | *UB* |
| *Ratio 2003-2006* | | | | | | | | | | | | | | | | | | | | | | | | |
| *EMC* | 0.8850 | 0.9330 | 0.8791 | 0.9435 | 0.8674 | 0.9627 | 0.9046 | 0.9481 | 0.8999 | 0.9568 | 0.8903 | 0.9776 | 0.8909 | 0.9392 | 0.8836 | 0.9479 | 0.8702 | 0.9680 | 0.8891 | 0.9327 | 0.8851 | 0.9404 | 0.8774 | 0.9537 |
| *EC* | 1.0009 | 1.0507 | 0.9951 | 1.0583 | 0.9829 | 1.0713 | 0.9959 | 1.0407 | 0.9905 | 1.0469 | 0.9815 | 1.0572 | 0.9987 | 1.0460 | 0.9916 | 1.0532 | 0.9803 | 1.0676 | 0.9943 | 1.0369 | 0.9889 | 1.0432 | 0.9798 | 1.0547 |
| *TC* | 0.8544 | 0.9113 | 0.8472 | 0.9228 | 0.8323 | 0.9459 | 0.8814 | 0.9355 | 0.8739 | 0.9464 | 0.8617 | 0.9680 | 0.8627 | 0.9195 | 0.8560 | 0.9290 | 0.8392 | 0.9517 | 0.8678 | 0.9123 | 0.8621 | 0.9179 | 0.8503 | 0.9312 |
| *PEC* | 1.0018 | 1.0340 | 0.9974 | 1.0383 | 0.9892 | 1.0483 | 0.9988 | 1.0254 | 0.9957 | 1.0290 | 0.9893 | 1.0354 | 0.9997 | 1.0318 | 0.9949 | 1.0362 | 0.9885 | 1.0444 | 0.9959 | 1.0217 | 0.9925 | 1.0253 | 0.9870 | 1.0315 |
| *SEC* | 0.9918 | 1.0244 | 0.9870 | 1.0286 | 0.9800 | 1.0371 | 0.9917 | 1.0209 | 0.9878 | 1.0251 | 0.9810 | 1.0332 | 0.9914 | 1.0212 | 0.9872 | 1.0260 | 0.9792 | 1.0341 | 0.9928 | 1.0208 | 0.9893 | 1.0250 | 0.9820 | 1.0320 |
| *Ratio 2006-2009* | | | | | | | | | | | | | | | | | | | | | | | | |
| *EMC* | 1.0755 | 1.1519 | 1.0623 | 1.1606 | 1.0376 | 1.1749 | 1.0780 | 1.1454 | 1.0652 | 1.1538 | 1.0410 | 1.1643 | 1.0770 | 1.1545 | 1.0636 | 1.1616 | 1.0388 | 1.1744 | 1.1024 | 1.1539 | 1.0932 | 1.1603 | 1.0751 | 1.1711 |
| *EC* | 0.9587 | 1.0058 | 0.9512 | 1.0130 | 0.9405 | 1.0269 | 0.9680 | 1.0104 | 0.9618 | 1.0171 | 0.9505 | 1.0304 | 0.9610 | 1.0080 | 0.9545 | 1.0143 | 0.9411 | 1.0289 | 0.9658 | 1.0080 | 0.9604 | 1.0147 | 0.9511 | 1.0249 |
| *TC* | 1.0888 | 1.1872 | 1.0716 | 1.1982 | 1.0423 | 1.2156 | 1.0823 | 1.1709 | 1.0670 | 1.1819 | 1.0408 | 1.2004 | 1.0890 | 1.1860 | 1.0719 | 1.1974 | 1.0409 | 1.2201 | 1.1153 | 1.1807 | 1.1038 | 1.1893 | 1.0775 | 1.2056 |
| *PEC* | 0.9778 | 1.0071 | 0.9734 | 1.0112 | 0.9668 | 1.0196 | 0.9898 | 1.0137 | 0.9863 | 1.0177 | 0.9795 | 1.0247 | 0.9773 | 1.0070 | 0.9726 | 1.0107 | 0.9659 | 1.0191 | 0.9911 | 1.0149 | 0.9875 | 1.0187 | 0.9811 | 1.0251 |
| *SEC* | 0.9744 | 1.0053 | 0.9695 | 1.0104 | 0.9625 | 1.0191 | 0.9732 | 1.0018 | 0.9690 | 1.0056 | 0.9615 | 1.0133 | 0.9768 | 1.0075 | 0.9720 | 1.0117 | 0.9652 | 1.0197 | 0.9704 | 0.9978 | 0.9664 | 1.0015 | 0.9605 | 1.0090 |
| *Ratio 2003-2009* | | | | | | | | | | | | | | | | | | | | | | | | |
| *EMC* | 0.9874 | 1.0231 | 0.9808 | 1.0279 | 0.9703 | 1.0379 | 0.9981 | 1.0309 | 0.9928 | 1.0363 | 0.9804 | 1.0484 | 0.9922 | 1.0268 | 0.9853 | 1.0312 | 0.9745 | 1.0416 | 1.0026 | 1.0221 | 0.9986 | 1.0247 | 0.9900 | 1.0304 |
| *EC* | 0.9936 | 1.0135 | 0.9908 | 1.0162 | 0.9862 | 1.0219 | 0.9944 | 1.0127 | 0.9918 | 1.0153 | 0.9874 | 1.0202 | 0.9931 | 1.0129 | 0.9903 | 1.0159 | 0.9856 | 1.0209 | 0.9923 | 1.0097 | 0.9898 | 1.0122 | 0.9862 | 1.0174 |
| *TC* | 0.9800 | 1.0231 | 0.9727 | 1.0289 | 0.9612 | 1.0426 | 0.9908 | 1.0315 | 0.9847 | 1.0371 | 0.9732 | 1.0504 | 0.9848 | 1.0271 | 0.9777 | 1.0321 | 0.9649 | 1.0427 | 0.9973 | 1.0231 | 0.9925 | 1.0265 | 0.9824 | 1.0334 |
| *PEC* | 0.9983 | 1.0113 | 0.9964 | 1.0131 | 0.9935 | 1.0171 | 1.0013 | 1.0123 | 0.9998 | 1.0137 | 0.9966 | 1.0160 | 0.9973 | 1.0102 | 0.9954 | 1.0119 | 0.9908 | 1.0159 | 1.0006 | 1.0113 | 0.9992 | 1.0128 | 0.9960 | 1.0157 |
| *SEC* | 0.9924 | 1.0049 | 0.9909 | 1.0070 | 0.9874 | 1.0104 | 0.9908 | 1.0028 | 0.9894 | 1.0045 | 0.9863 | 1.0075 | 0.9928 | 1.0055 | 0.9913 | 1.0073 | 0.9883 | 1.0109 | 0.9896 | 1.0010 | 0.9881 | 1.0027 | 0.9853 | 1.0062 |

## 5. Conclusions

Malmquist Index methodology is widely used in the literature with the aim of measuring the productivity growth within two time periods as the distance between each DMU and the frontier for each period. This methodology allows to decomposing the productivity change into three components: the technical efficiency, the scale efficiency and the technology change. To perform a Malmquist index we need a panel database.

In this paper, we propose a new Educational Malmquist in order to compare productivity divergences between public and government dependent private schools across the time when only a pseudo-panel database is available. For this purpose we use Basque Country school data from PISA 2003, 2006 and 2009 that provide us wide information about the educational context.

The results show that GDPS outperform PS within the 2003-2009, using different alternatives to matching school samples, although the productivity difference among both school types reduces within this time period. However, it is generally observed a productivity divergence peak for GDPS in 2006 related to previous and following years. On the other hand, the productivity evolution within 2003-2009 benefits PS, as a consequence of the relative productivity loss for GDPS during this period, although this result is not significant. Moreover, the technology gap is the main component in the Educational Malmquist, especially for alternative 2 and 4, which are significant at 99%.

Finally, our results seem to indicate that more similar average family characteristics among schools will increase the productivity in the educational Basque Country system. Nevertheless, these conclusions should be interpreted cautiously, since they are referred to a particular context and time however their implications are very relevant for the design of educational-policy in the Basque Country.

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